WLF 419 - Waterfowl and Wetlands Ecology and Management
Lecture 14 – Elasticity Analysis
Next Time – Population Surveys

In our discussion of breeding parameters and survival probability we noted extensive spatial, temporal, and taxonomic variation in demographic parameters. Managers are faced with a daunting task of identifying which parameter(s) should be targeted for. One approach to identify the most important parameters is elasticity analysis. We will examine the general approach to elasticity analysis, consider a specific example with Greater Snow geese, and discussion general patterns across the range of waterfowl taxa.

Population Dynamics

$$N_{t+1} = N_t + B + I - D - E$$

- Matrix Modeling

$$\begin{matrix} F_1 & F_2 & F_3 \\ S_1 & 0 & 0 \\ 0 & S_2 & S_3 \end{matrix} \quad \times \quad \begin{matrix} N_{1,t} \\ N_{2,t} \\ N_{3,t} \end{matrix} \quad = \quad \begin{matrix} N_{1,t+1} \\ N_{2,t+1} \\ N_{3,t+1} \end{matrix}$$

Projection Matrix
pre- and post-birth pulse

Abundance @ t

Abundance @ t+1

- Lambda
  - Finite rate of population change
    - Changes in Abundance
    - Matrix Characteristics

Retrospective and Prospective Analysis

- Which population parameter has the most effect on past variation in population dynamics (lambda)?
  - Life Table Response Experiments – Asymptotic Conditions
  - Lifestage Simulation Analysis - Asymptotic Conditions
  - Reverse Capture Histories

- Which parameter would you change to have the most future impact on population dynamics?
    - Analytical Elasticities – Asymptotic Conditions

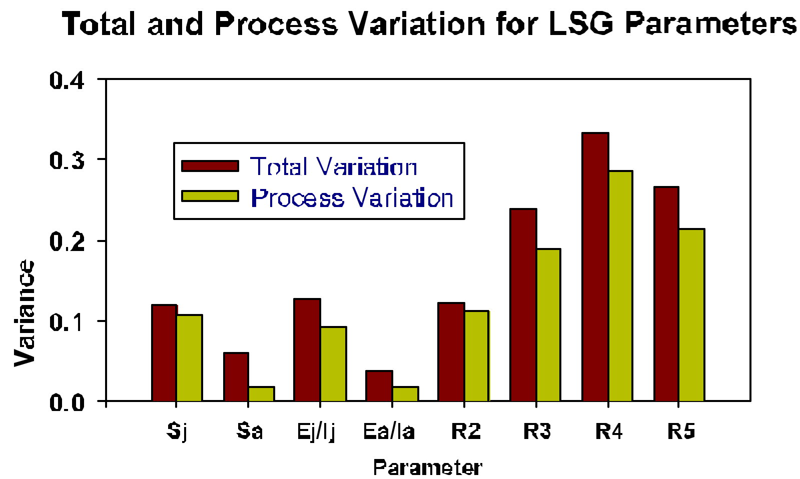
F_1 F_2 F_3
S_1 0 0
0 S_2 S_3 ➡)8

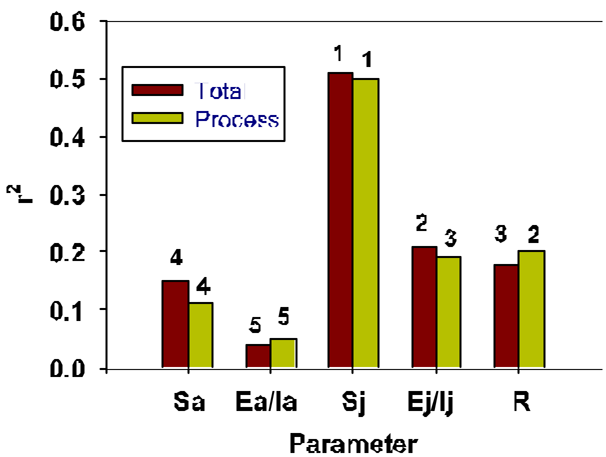
- Brute Force – Asymptotic Conditions

Retrospective Studies

- Lesser Snow Geese
    - La Perouse Bay
    - 30+ years of data

FIG. 1. Major nesting areas of mid-continent Lesser Snow Geese in northeastern Canada. Sizes of circles are proportional to estimated numbers of geese breeding at each colony in 1997 or 1998: Queen Maud Gulf, 1.4 × 10^6; Baffin Island, 1.8 × 10^6; Southampton Island, 0.7 × 10^6; Cape Henrietta Maria, 0.2 × 10^6; McConnell River, 0.15 × 10^6; La Pérouse Bay, 16–22 × 10^3.





From Mills and Lindberg (2001)

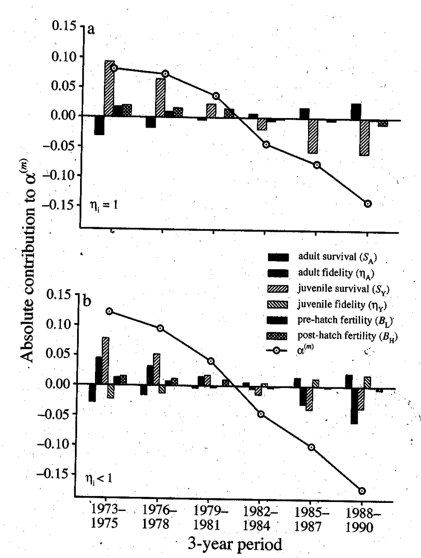


FIG. 8. Lower-level contributions of individual vital rates to variation in $\alpha^{(m)}$ for each treatment block, for models (a) without fidelity rate (i.e., $\eta = 1$) and (b) with emigration (fidelity) rate (i.e., $\eta < 1$). Bars represent the proportion of the contribution of each rate to $\alpha^{(m)}$. Since neither breeding propensity (γ_x) nor immigration rate (I) varies over time (Table 1), neither contributes to variation in I.

From Cooch et al. (2001)

- Mid-continent Mallards

Table 2. Estimates of the mean value, empirical SD, SD of process variation (σ), and 90% CI of $\hat{\sigma}$ for parameters of a demographic model of female mid-continent mallards and for annual survival of female mid-continent mallards.

	Parameter Estimates				Data	
Parameter	Mean	Empirical SD	$\hat{\sigma}$	90% CI of $\hat{\sigma}$	n	Source(s)
Clutch Size[a]	4.55	Uniform distribution 3.55 - 5.55				Bellrose 1976
Egg Hatch	0.91	N/A	0.050	N/A		Johnson et al. 1992
Nest Success[b]	0.13	0.089	0.075[c]	0.058 - 0.101	42	Klett et al. 1988; Greenwood et al. 1995; IWWR, unpublished data
Duckling Survival	0.39	0.134	0.110[c]	0.075 - 0.177	14	Rotella and Ratti 1992; IWWR, unpublished data; Hestbeck et al. 1987
Breeding Incidence[d]	0.968	0.040	0.024[c]	0.009 - 0.053	11	IWWR, unpublished data
Renesting Intensity[e]	-0.26	0.106	0.097[c]	0.069 - 0.164	11	IWWR, unpublished data
Breeding Survival	0.72	0.071	0.067[c]	0.051 - 0.095	19	IWWR, unpublished data
Non-breeding Survival	0.80	N/A	0.051[f]	0.00 - 0.088		derived from Breeding and annual survival
Annual Survival[g]	0.58	0.192	0.065[c]	0.044 - 0.086	313	Chu and Hestbeck 1989

[a]Clutch Size differed between stage classes. Means for SY and ASY females were 4.30 and 4.80.

[b]Nest Success estimated from all nests. Values of Nest Success used in sensitivity analyses were increased by 10% so that Nest Success was independent of Breeding Survival. See text for additional explanation.

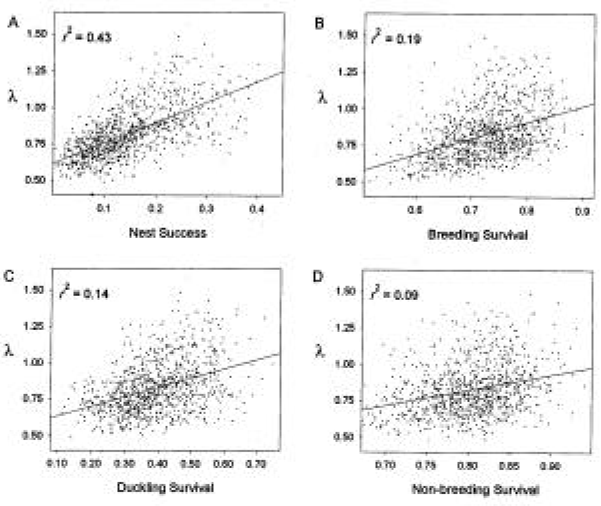
[c]SD of process variation estimated using weighted estimator (see text for details).

[d]Breeding Incidence differed between stage classes. Means for SY and ASY females were 0.945 and 0.984.

[e]Renesting Intensity differed between stage classes. Means for SY and ASY females were -0.28 and -0.23.

[f]SD of process variation estimated using bootstrap procedure (see text for details).

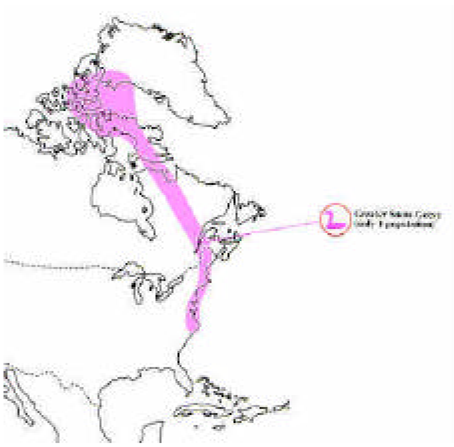
[g]Annual survival was not a model parameter, but was used in conjunction with Breeding Survival to derive parameter estimates for Non-breeding Survival.



From Hoekman et al. (unpublished)

Prospective Analysis

- Greater snow geese and certain population of lesser snow geese have experienced exponential population growth during the last 2 or 3 decades (Fig. 4-1).



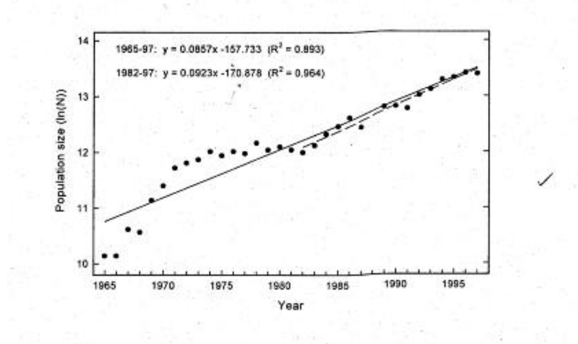


Figure 4-1. Growth of the greater snow goose population, 1965-1997. The slope of the relationship is an estimate of r, the annual rate of increase. The regression is calculated for the whole period (1965-1997) and for the period of most recent increase (1982-1997).

From Gauthier and Brault (1998)

- The annual finite rate of increase (8) for 1982-1997 is 1.09
- This increase in snow geese has created a management dilemma
  - geese are degrading habitats at staging and wintering areas (greater snow geese)
  - and in arctic breeding areas (lesser snow geese)

- To explore management options with a matrix approach Gauthier and Brault (1998) considered a post-birth pulse model with 4 age classes: juveniles, 2-year-olds, 3-years-olds, and 4+-year-olds (Figure 4.2)

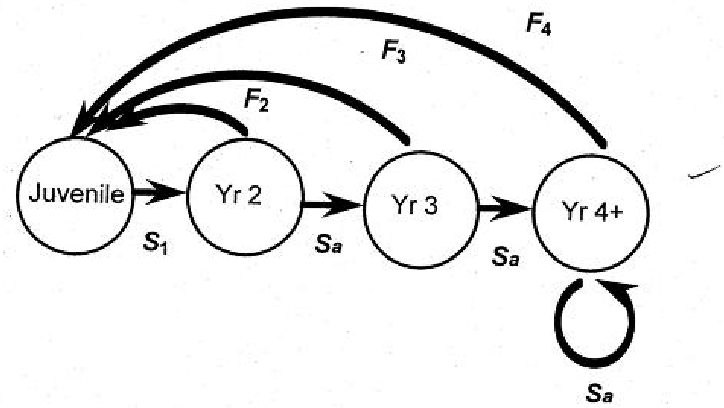


Figure 4-2. Life cycle of the greater snow goose used for the model. F = fecundity, S = survival. Numbers refer to stage, each stage referring to an age class (or year), starting at 1 month of age (i.e. at banding). Thus, Stage 1 of the model (Juvenile) extends from fledging (~1 month of age) until ~1.1 years old. See text for details. (S_1 = 1st year survival, S_a = adult survival).

- Parameter Estimates

Table 4-1. Definition of reproductive and survival parameters.

Parameter	Definition
Breeding propensity (BP)	Probability that a female alive at time t will attempt to breed.
Total clutch laid (TCL)	Total number of eggs laid by a nesting female (clutch size).
Nesting success (NS)	Probability that a nest will be successful, i.e. that at least one egg will hatch.
Egg survival ($P1$)	Probability that an egg will survive to hatching in successful nests.
Hatching success ($P2$)	Probability that an egg that reached hatching stage in successful nests will produce a gosling leaving the nest.
Gosling survival ($P3$)	Probability that a gosling leaving the nest survives until banding, which occurs just prior to fledging.
1st year survival (S_1)	Probability that a bird will survive from banding at ~1 month to the next banding period, at ~1.08 years.
Annual adult survival (S_a)	Probability that an adult bird (> 1 year old) will survive from one banding period to the next.

Table 4-2. Estimates of reproductive and survival parameters for greater snow geese. All values are estimated from Bylot Island except breeding propensity which are values taken from LaPérouse Bay (Rockwell et al. 1997). Note that breeding propensity and adult survival were set equal in good, average and bad years.

Parameter	Stage (age class)	Good year	Average year	Bad year
Breeding propensity (BP)	1 2 3 4		0 0.35 0.77 0.88	
Total clutch laid (TCL)	2 – 4	4.44	3.81	3.39
Nesting success (NS)	2 – 4	0.89	0.71	0.34
Egg survival ($P1$)	2 – 4	0.956	0.905	0.885
Hatching success ($P2$)	2 – 4	0.968	0.930	0.879
Gosling survival ($P3$)	2 – 4	0.739	0.705	0.529
Survival (S)	1 2-4	0.50	0.35 0.83	0.12

F_i (i = 2- 4) = S_a x BP_i x (TCL/2) x NS_i x P1 x P2 x P3
so
F_2 = 0.83 x 0.35 x (3.81/2) x 0.71 x 0.905 x 0.930 x 0.705 = 0.233
TCL is divided by 2 because we are only modeling females and we assume a 50:50 age ratio in embryos.
F_3 = 0.513
F_4 = 0.566

- Using matrix projections and analytical sensitivity analysis they determined elastic ities of vital rates for bad, average, and good years (Fig. 4-3)

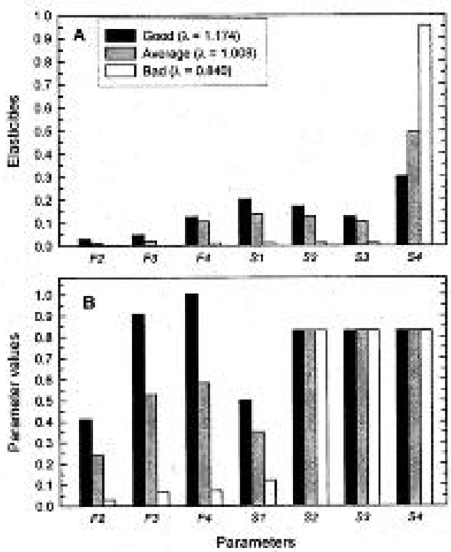


Figure 4-3. Elasticities (A) and values (B) of parameters used in the model for good, average and bad years of reproduction. *F* = fertility, *S* = survival. Numbers refer to stages (= age classes; see Fig. 4-2).

Elasticities

• Expected proportionate change in 8 with a proportionate change in vital rate
• infinitesimally small change in vital rate, one vital rate at a time
• Stable Age Distribution
• Elasticities sum to 1.00
• Thus, proportionate change in a vital rate with an elasticity of 0.3 would change 8 3x as much as the same proportionate change in a vital rate with an elasticity of 0.1
• Sensitivities are used to evaluate absolute changes in parameters

For Example

Parameter	Elasticity	Sensitivity	Value	10% Relative Increase	10% Absolute Increase
S_4	0.500	0.6036	0.830	0.913	0.930
F_2	0.011	0.0473	0.233	0.256	0.333

A 10% relative increase in S_4 results in about 45x more increase in 8 than a 10% relative increase in F_2. A 10% absolute increase in S_4 results in about 12x more increase in 8 than a 10% absolute increase in F_2.

- Mid-Continent Mallards

Table 4. Analytic sensitivities and elasticities of parameters for a demographic model of female mid-continent mallards at mean values of parameters and across process variation in parameters .

Parameter	Analytic Sensitivity[a]		Analytic Elasticity[b]	
	mean[c]	SD[d]	mean[c]	SD[d]
Clutch Size	0.05	0.04	0.29	0.12
Egg Hatch	0.27	0.18	0.29	0.11
Nest Success	1.66	0.79	0.26	0.09
Duckling Survival	0.62	0.68	0.29	0.11
Breeding Incidence	0.21	0.25	0.24	0.13
Renesting Intensity	0.44	0.61	0.14	0.04
Breeding Survival	1.13	0.24	1.00	0.00
Non-breeding Survival	1.06	0.23	1.00	0.00

[a]Relative $\Delta\lambda$ resulting from an absolute and infinitesimal change in a vital rate when others were held constant.

[b]Relative $\Delta\lambda$ resulting from a proportional and infinitesimal change in a vital rate when others were held constant.

[c] From a model composed of mean values of parameters.

[d]SD from simulations of 5,000 replicates with process variation in parameters.

From Hoekman et al. (unpublished)

Elasticities based on Life-histories

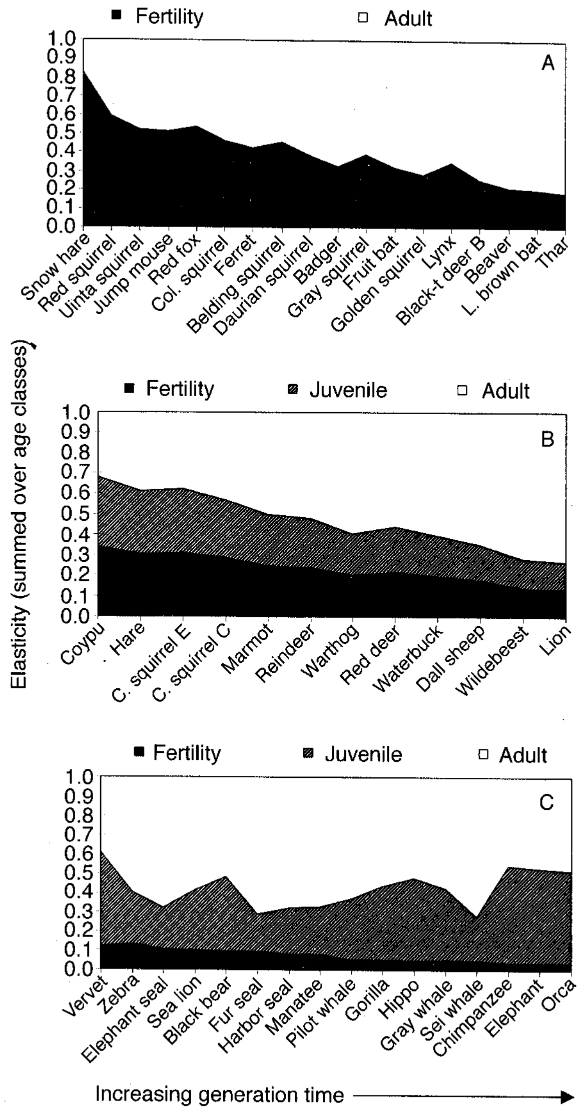


FIG. 2. Area plots showing the stage-specific elasticities for each mammal population, grouped by age at maturity and ordered by increasing generation time. (A) Age at first maturity = 1 yr; no juvenile stage. (B) Age at first maturity = 2 yr; fertility elasticity = juvenile survival elasticity (see *Methods*). (C) Age at first maturity >2 yr.

From Heppell et al. (2000)

Literature Cited

Heppell, S.S., H. Caswell, and L.B. Crowder. 2001. Life histories and elasticity patterns: perturbation analysis for species with minimal demographic data. Ecology 81:654-665.

Hoekman, S.T., L.S. Mills, D.W. Howerter, J.H. DeVries, and I.J. Ball. In Press. Sensitivity analysis of the life cycle of mid-continent mallards. Journal of Wildlife Management.

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